清华大学

综合论文训练

题目：Folding and 1-Cut 问题的求解与可视化

系　　别：计算机科学与技术系

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摘　要

由于涉及到平面图骨架结构、三角化、凸分解等计算几何中重要的概念和方法，传统纸艺近年来一直是计算几何中的一个研究热点，甚至形成了一个称之为 Computational Origami 的专门研究方向. 在本文中，我们研究了 Computational Origami 中的一个著名问题 Folding and Cutting—的求解与可视化算法及其实现. 我们首先引入一种新的平面图骨架结构——直架，并介绍了它的 $O(n^2 \log n)$ 生成算法. 直架本身是不可折叠的，因此又在直架的基础上引入了垂线折痕. 经过适当的内外折分配，两者共同组成了折痕图. 为了模拟凸多边形的 3 维折叠过程，我们考察了折叠极基的问题，并由此得到了一个递推求解的模拟方法.

关键词

计算几何，纸艺，直架，垂线折痕，折痕图，可视化，极基

Abstract

Since it has deep relations with so many important concepts and methods in the computational geometry literature, such as skeletons of planar graph, triangulation and convex decomposition, traditional Origami has always been a hotspot in recent years, and even forms a special field called Computational Origami. In this paper, we study the solution and visualization algorithms for a famous problem in this field, Folding and Cutting. Firstly, we introduce a novel type of skeleton for planar graphs, Straight Skeleton, and also present an $O(n^2 \log n)$ algorithm for finding the straight skeletons of simple polygons. Straight skeleton by itself is not foldable. Therefore, another type of crease, perpendicular fold, is added to solve the problem. After proper mountain-valley assignment, straight skeleton and perpendicular folds build up the final crease pattern. By investigating the extreme bases, we get a recursive method to simulate the 3D folding process of convex polygons.

Keywords

Computational Geometry, Origami, Straight Skeleton, Perpendicular Folds, Crease Pattern, Visualization, Extreme Base

目　录

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附录一　名词索引

Corridor　走廊

Crease Pattern　折痕图

Cut Edge　切边

Distance-Preserving　保距

Edge Event　边消失事件

Extreme Origami Base　折纸极基

Flap　折翼

Flat Foldable 可折平的

Flip Event　边交换事件

Inner Wavefront　内向波阵面

Medial Axis　中轴

Monotone Polygon　单调多边形

Mountain Fold　外折

Origami　折纸

Outer Wavefront　外向波阵面

Perpendicular Edge　垂线边

Perpendicular Fold　垂线折痕

Real Perpendicular　实垂线

Reflex Vertex　凹顶点(或优角顶点)

Shadow Tree　阴影树

Skeleton Face　直架面

Split Event　边分裂事件

Straight Skeleton　直架

Unbounded Triangle　无限三角形

Uniaxial Base　单轴基

Valley Fold　内折

Wavefront　波阵面